

LOGARITHM & QUADRATIC EQUATION

3. Let x satisfies the equation $\log_3(\log_9 x) = \log_9(\log_3 x)$ then the product of the digits in x is

(A) 9

(B) 18

(C) 36

(D) 8

$$8 \times 1 = 8$$

$$\log_3\left(\frac{\log_3 x}{2}\right) = \frac{\log_3(\log_3 x)}{2}$$

$$\log\left(\frac{t}{2}\right) = \frac{\log_3 t}{2} \Rightarrow 2(\log_3 t - \log_3 2) = \log_3 t$$

$$\log_3 t = 2 \log_3 2 = \log_3 4 \Rightarrow t = 4 \Rightarrow \log_3 x = 4$$

$$x = 3^4 = 81$$

LOGARITHM & QUADRATIC EQUATION

4. If $\log_2(\log_3(\log_4 x)) = 0$, $\log_4(\log_3(\log_2 y)) = 0$ and $\log_3(\log_4(\log_2 z)) = 0$, then the correct option is

(A) $x > y > z$

(B) $x > z > y$

(C) $z > x > y$

(D) $z > y > x$

$$\begin{aligned} \log_3(\log_4 x) &= 2^0 = 1 \\ \log_4 x &= 3^1 = 3 \\ x &= 4^3 = 64 \end{aligned}$$

$$\begin{aligned} \log_3(\log_2 y) &= 1 \\ \log_2 y &= 3 \\ y &= 2^3 = 8 \end{aligned}$$

$$\begin{aligned} \log_4(\log_2 z) &= 1 \\ \log_2 z &= 4 \\ z &= 2^4 = 16 \end{aligned}$$

$$x > z > y$$

LOGARITHM & QUADRATIC EQUATION

5. If $\log_3(\log_2 a) + \log_{\frac{1}{3}}(\log_{\frac{1}{2}} b) = 1$, then the value of ab^3 is

(A) 9

(B) 3

~~(C) 1~~

(D) $\frac{1}{3}$

$$\log_3(\log_2 a) + \log_{3^{-1}}(\log_{2^{-1}} b) = 1$$

$$\log_3(\log_2 a) - \log_3(-\log_2 b) = 1$$

$$\log_3(\log_2^A a) - \log_3(\log_2^B \frac{1}{b}) = 1$$

$$\log_3\left(\frac{\log a}{\log_2 b}\right) = 1$$

$$\frac{\log_2 a}{\log_2 \frac{1}{b}} \xrightarrow{3}$$

$$\log_2 a = 3 \log_2 b^{-1}$$

$$\log_2 a = \log_2 b^{-3}$$

$$a = \frac{1}{b^3} \Rightarrow ab^3 = 1$$

LOGARITHM & QUADRATIC EQUATION

6. If $\log(x + y) = \log 2 + \frac{1}{2}\log x + \frac{1}{2}\log y$, then

(A) $x + y = 0$ ✗

(B) $xy = 1$ ✗

(C) $x^2 + xy + y^2 = 0$

(D) $x - y = 0$ ✓✓

$$\log(x+y) = \log 2 + \log \sqrt{x} + \log \sqrt{y}$$

$$\log(x+y) = \log 2\sqrt{xy}$$

$$x+y = 2\sqrt{xy}$$

$$x+y - 2\sqrt{x}\sqrt{y} = 0$$

$$(\sqrt{x} - \sqrt{y})^2 = 0$$

$$\sqrt{x} - \sqrt{y} = 0 \Rightarrow x = y$$

LOGARITHM & QUADRATIC EQUATION

7. $\frac{1}{\log_{\sqrt{bc}} abc} + \frac{1}{\log_{\sqrt{ca}} abc} + \frac{1}{\log_{\sqrt{ab}} abc}$ has the value equal to

(A) $1/2$

(B) 1

(C) 2

(D) 4

$$\log_{abc} \sqrt{bc} + \log_{abc} \sqrt{ca} + \log_{abc} \sqrt{ab}$$

$$\log_{abc} (\sqrt{bc} \times \sqrt{ca} \times \sqrt{ab}) = \log_{abc} abc = 1$$

LOGARITHM & QUADRATIC EQUATION

8. If $5x^{\log_2 3} + 3^{\log_2 x} = 162$ then logarithm of x to the base 4 has the value equal to

(A) 2

(B) 1

(C) -1

(D) $\frac{3}{2}$

$$5x^{\log_2 3} + 3^{\log_2 x} = 162$$

$$5 \cdot 3^{\log_2 x} + 3^{\log_2 x} = 162$$

$$5t + t = 162$$

$$6t = 162 \Rightarrow t = 27$$

$$3^{\log_2 x} = 3^3$$

$$\log_2 x = 3$$

$$x = 2^3 = 8$$

$$\log_4 8$$

$$= \frac{\log_2 8}{\log_2 4}$$

$$= \frac{3}{2} \log_2 2^2$$

$$= \frac{3}{2}$$

LOGARITHM & QUADRATIC EQUATION

9. The value of $\log_{\frac{4}{3}} \left(\frac{56 + \sqrt{56 + \sqrt{56 + \sqrt{56 + \dots \infty}}}}{\sqrt{64 \sqrt{64 \sqrt{64 \dots \infty}}}} \right)$ is equal to
- (A) 0 (B) 2 (C) 3 (D) 4

$$x = 56 + \sqrt{56 + \sqrt{56 + \sqrt{56 + \dots \infty}}}$$

$$x = 56 + \sqrt{x}$$

$$x - \sqrt{x} - 56 = 0$$

$$t^2 - t - 56 = 0$$

$$(t - 8)(t + 7) = 0$$

$$t = 8 \quad t = -7$$

$$\sqrt{x} = 8 \quad \text{ⓧ}$$

$$y = \sqrt{64 \sqrt{64 \sqrt{64 \dots \infty}}}$$

$$y = \sqrt{64y} \Rightarrow \sqrt{y} = 8 \Rightarrow y = 64$$

LOGARITHM & QUADRATIC EQUATION

10.

	Column-I		Column-II
(A)	$\log_2 x = -\log_2 7$, then the value of x is	(P)	$\frac{1}{49}$
(B)	$\log_8 y = \frac{-1}{\log_3 2}$, then the value of y is	(Q)	$\frac{1}{36}$
(C)	$\log_{\frac{1}{4}} z = \log_2 6$, then the value of z is	(R)	$\frac{1}{27}$
(D)	$\log_{\frac{1}{9}} w = \log_3 7$, then the value of w is	(S)	7
		(T)	$\frac{1}{6}$

$\log_8 y = -\log_2 7$
 $\frac{1}{3} \log_2 y = 2 \log_2 \frac{1}{3}$
 $\log_2 y^{\frac{1}{3}} = \log_2 \frac{1}{3}$
 $y^{\frac{1}{3}} = \frac{1}{3}$
 $y = \frac{1}{27}$

DPP-2

$$4) \quad x^{\log_{10} x} = \left(100 + 2^{\sqrt{\log_2 3}} - 3^{\sqrt{\log_3 2}}\right) x$$

$$\boxed{a^{\sqrt{\log_c b}} = b^{\sqrt{\log_c a}}}$$

mistake

$$\log_{10} x^{\log_{10} x} = \log_{10} (100x)$$

$$\boxed{\log_{10} x} \log_{10} x = 2 + \boxed{\log_{10} x}$$

$$t^2 - t - 2 = 0$$

$$(t-2)(t+1) = 0$$

$$t = 2$$

$$\log_{10} x = 2$$

$$x = 100$$

$$t = -1$$

$$\log_{10} x = -1$$

$$x = \frac{1}{10}$$

$$100 \times \frac{1}{10} = 10$$

$$3) \quad a_n = \frac{1}{\log_n 2002} = \log_{2002} n$$

$$b = a_2 + a_3 + a_4 + a_5$$

$$= \log_{2002} 2 + \log_{2002} 3 + \log_{2002} 4 + \log_{2002} 5$$

$$b = \log_{2002} (2 \times 3 \times 4 \times 5)$$

$$c = a_{10} + a_{11} + a_{12} + a_{13} + a_{14}$$

$$c = \log_{2002} (10 \times 11 \times 12 \times 13 \times 14)$$

$$\begin{array}{r} 14 \\ 14 \\ \hline 154 \\ 13 \\ \hline 562 \\ 159 \\ \hline \end{array}$$

$$b - c = \log_{2002} \left\{ \frac{2 \times 3 \times 4 \times 5}{10 \times 11 \times 12 \times 13 \times 14} \right\}$$

$$= \log_{2002} (2002)^{-1} = -1$$

$$\begin{aligned}
 5) \quad L &= \sum_{r=1}^{2400} \log_7 \left(\frac{r+1}{r} \right) = \log_7 \left\{ \frac{8}{7} \times \frac{9}{8} \times \frac{10}{9} \times \frac{11}{10} \times \dots \times \frac{2401}{2400} \right\} \\
 &= \log_7 \left\{ \frac{2401}{7} \right\} = 3
 \end{aligned}$$

$$\begin{aligned}
 M &= \prod_{r=2}^{1023} \log_r(r+1) = \frac{\log 3}{\log 2} \times \frac{\log 4}{\log 3} \times \frac{\log 5}{\log 4} \times \frac{\log 6}{\log 5} \times \dots \times \frac{\log(1024)}{\log 1023} \\
 &= \frac{\log 1024}{\log 2} = \log_2 2^{10} = 10
 \end{aligned}$$

$$N = \sum_{r=2}^{2011} \frac{1}{\log_r P} = \sum_{r=2}^{2011} \log_P r = \log_P \left\{ \underbrace{2 \times 3 \times 4 \times \dots \times 2011}_{r=1} \right\} = \log_P P = 1$$

$$(7) \quad P = \log_5(\log_5 3) \quad \& \quad 3^{(c + 5^{-P})} = 405 \Rightarrow 3^c \cdot 3^{5^{-P}} = 405$$

$$5^{-P} = 5^{\log_5(\log_5 3)}$$

$$= (\log_5 3)^{-1} = \frac{1}{\log_5 3} = \boxed{\log_3 5}$$

$$3^c \times 3^{\log_3 5}$$

$$3^c \times 5 = \frac{81}{405} = 3^4$$

$$c = 4$$

Q9 $\frac{a + \log_4 3}{a + \log_2 3} = \frac{a + \log_8 3}{a + \log_4 3} = b$

$$\frac{2}{4} = \frac{6^2}{8^3}$$

No

$$\frac{2}{4} = \frac{6}{12} = \frac{6-2}{12-4} = \frac{4}{8}$$

$$\frac{(a + \log_8 3) - (a + \log_4 3)}{(a + \log_4 3) - (a + \log_2 3)} = b$$

$$\frac{\frac{1}{3} \log_2 3 - \frac{1}{2} \log_2 3}{\frac{1}{2} \log_2 3 - \log_2 3} = \frac{\left(\frac{1}{3} - \frac{1}{2}\right)}{-\frac{1}{2}} = \frac{+\frac{1}{6}}{+\frac{1}{2}} = \frac{1}{3}$$

Quadratic Eqⁿ

Polynomial f_{xn}

$$f(x) = \boxed{a_0}x^n + a_1x^{n-1} + a_2x^{n-2} + \dots + a_{n-2}x^2 + \underline{a_{n-1}x + a_n} \quad \text{is Poly f_{xn}}$$

$$\begin{aligned} 1) \quad & f(x) = a \rightarrow \text{constant} \\ & f(x) = ax + \boxed{b} \rightarrow \text{linear f_{xn}} \\ & f(x) = ax^2 + bx + \boxed{c} \rightarrow \text{Quad f_{xn}} \\ & f(x) = ax^3 + bx^2 + cx + \boxed{d} \rightarrow \text{Cubic f_{xn}} \end{aligned} \quad \left. \vphantom{\begin{aligned} f(x) = a \\ f(x) = ax + b \\ f(x) = ax^2 + bx + c \\ f(x) = ax^3 + bx^2 + cx + d \end{aligned}} \right\} \text{Poly f_{xn}}$$

$$2) \rightarrow \text{highest deg of Poly} = \text{Deg of Poly}$$

$$3) a_0 = \text{L.C.}$$

Q. NOTF in Poly.

A) $\sqrt{5}x^3 + 2x - 7$

B) $\frac{2}{7}x^2 - 6\sqrt{3}x + 7$

C) $3x^2 - 6x^{-1} + 5$

D) $3x^3 + \frac{2}{x^{2/3}} + 4x$

2) Zeros of Poly

Egn $\rightarrow$ Roots
fxn $\rightarrow$ Zeros

1) $x = \alpha$ is said to be zero of Poly if
 $f(\alpha) = 0$

$(x-1)(x-2) = 0$

$x=1$ or $x=2$

2) $f(x) = x^2 - 3x + 2$ Q. fxn.

$x^2 - 3x + 2 = 0 \rightarrow$ Q. Egn

$f(1) = 1^2 - 3 \times 1 + 2 = 0$

$f(2) = 2^2 - 3 \times 2 + 2 = 0$

1, 2 are zeros
of $f(x) = x^2 - 3x + 2$

3)

$y = x^2 - 3x + 2 = (x-1)(x-2)$



4) If Zeros of Poly are α, β, γ

then Poly-fxn will be

$$f(x) = a(x-\alpha)(x-\beta)(x-\gamma)$$

Ex: $\rightarrow$ If zeros of Poly are 1, 2, 3.

then $f(x) = a(x-1)(x-2)(x-3)$

(5) Fundamental Theorem of Algebra

If deg. of Poly fxn is $n \in \mathbb{N}$ then.

No of zeros will also be n

Ex: $\rightarrow f(x) = (x-4)^3(x-5)$

Zeros = 4, 4, 4, 5 } 4 deg Eqn

No of zeros = 4

Distinct Zeros = 4, 5

Q If zeros are 4, 4, 4, 5
 $f(x) = a(x-4)^3(x-5)$

$$R_K \div A) \quad f(x) = 3$$

deg of $f(x)$ is not defined.

$$B) \quad f(x) = \boxed{1x^3} \text{ Single term.}$$

Monomial

$$f(x) = 1x^3 - 2 \rightarrow 2 \text{ terms}$$

$\rightarrow$ Binomial.

$$C) \quad L.C. = 1 \Rightarrow \text{Monic Polynomial.}$$

(6) Poly Eqⁿ

If $y = f(x)$ is a Poly $f(x)$.

then $f(x) = 0$ will be poly Eqⁿ

$$y = \underbrace{x^3 - 3x^2 + 7}_{\rightarrow \text{Zero}} \rightarrow \text{Cubic } f(x).$$

$$x^3 - 3x^2 + 7 = 0 \rightarrow \text{Cubic Eqⁿ Roots}$$

7) Q. Eqn.

Perfect Sqⁿ $\Rightarrow$ Shri Dharam Singh

$$ax^2 + bx + c = 0 \text{ is Q. Eqn.}$$

$$a \left\{ x^2 + \left(\frac{b}{a} \right) x + \frac{c}{a} \right\} = 0$$

$$a \left\{ \left(x + \frac{b}{2a} \right)^2 - \left(\frac{b}{2a} \right)^2 + \frac{c}{a} \right\} = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-b \pm \sqrt{D}}{2a}$$

$$\left(x + \frac{b}{2a} \right)^2 = \frac{b^2}{4a^2} - \frac{c}{a}$$

$$\left(x + \frac{b}{2a} \right) = \pm \sqrt{\frac{b^2 - 4ac}{4a^2}}$$

A) $x^2 - 4x + 7$

$$\left(x - 2 \right)^2 - 2^2 + 7$$

$$\left(x - 2 \right)^2 + 3$$

$$13 - \frac{9}{4}$$

B) $x^2 + 3x + 13$

$$\left(x + \frac{3}{2} \right)^2 - \left(\frac{3}{2} \right)^2 + 13$$

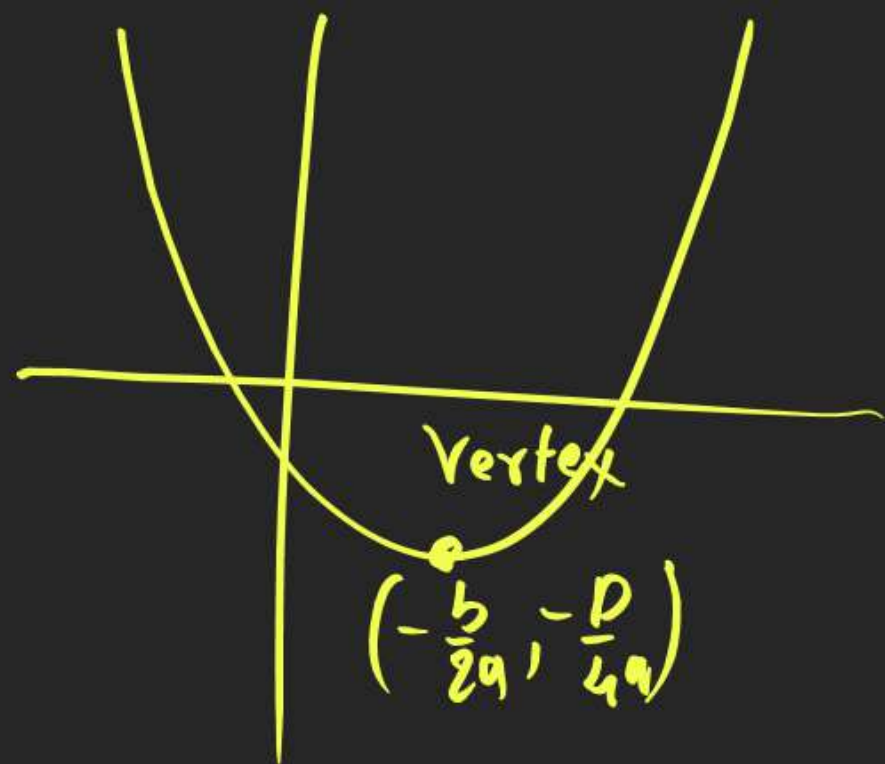
$$\left(x + \frac{3}{2} \right)^2 + \frac{43}{4}$$

$$x = \frac{-b \pm \sqrt{D}}{2a}$$

$$3x^2 - 7x + 2$$

$$= 3 \left\{ x^2 - \left(\frac{7}{3}\right)x + \frac{2}{3} \right\}$$

$$= 3 \left\{ \left(x - \frac{7}{6}\right)^2 - \left(\frac{7}{6}\right)^2 + \frac{2}{3} \right\}$$



Q. f(x) $\rightarrow$ graph.

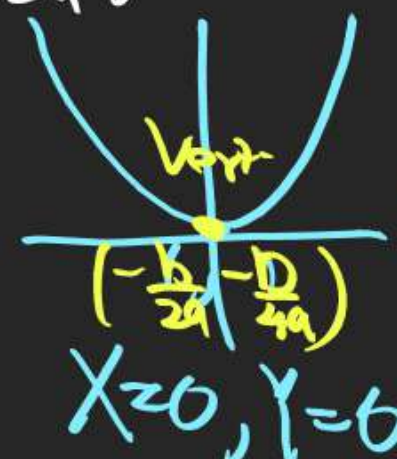
$$y = a \left(x^2 + \frac{b}{a}x + \frac{c}{a} \right)$$

$$y = a \left\{ \left(x + \frac{b}{2a} \right)^2 - \frac{b^2}{4a^2} + \frac{c}{a} \right\}$$

$$y = a \left(x + \frac{b}{2a} \right)^2 - \left(\frac{b^2 - 4ac}{4a} \right)$$

$$\left(y + \frac{D}{4a} \right) = a \left(x + \frac{b}{2a} \right)^2$$

$$Y = aX^2$$



$$\left(x + \frac{b}{2a}\right) = 0 \quad \left| \quad y + \frac{D}{4a} = 0 \right.$$

(8) Roots of Q. Eqn.

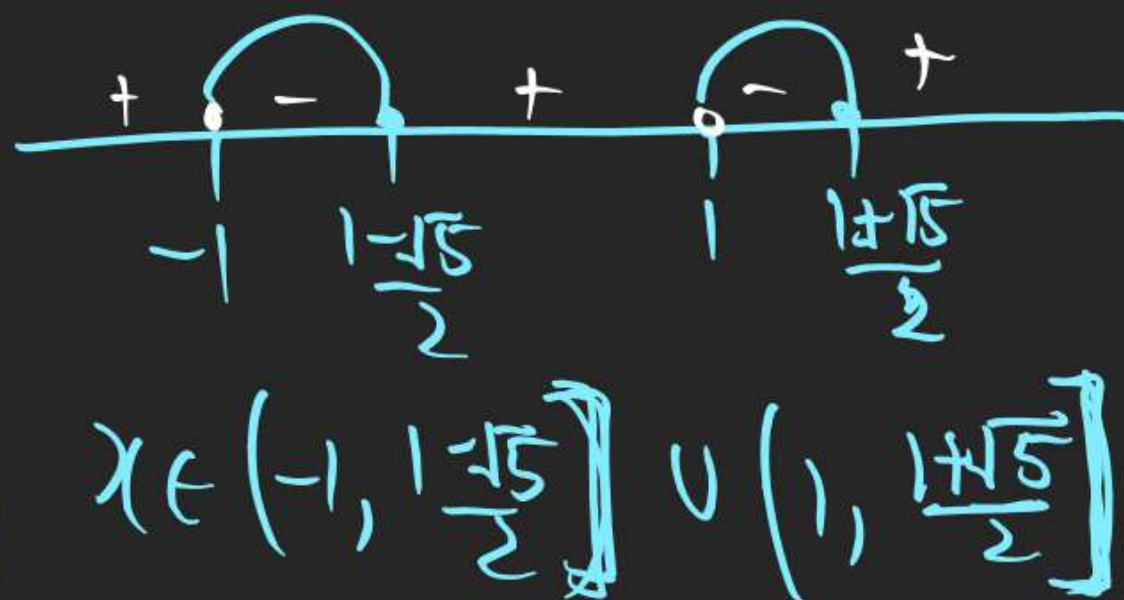
$$\chi = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \begin{cases} \nearrow -\frac{b + \sqrt{b^2 - 4ac}}{2a} = \alpha \\ \searrow -\frac{b - \sqrt{b^2 - 4ac}}{2a} = \beta \end{cases}$$

Q $2x^2 - 3x + 5 = 0$ find ΔR .

$$\chi = \frac{+3 \pm \sqrt{9 - 4 \times 2 \times 5}}{2 \times 2} = \frac{3 \pm \sqrt{-31}}{4} \rightarrow \text{Imaginary Roots}$$

Q $\frac{\chi^2 - \chi - 1}{\chi^2 - 1} \leq 0$ $\sqrt{5} = 2.2$

$$\frac{\left(\chi - \left(\frac{1 + \sqrt{5}}{2}\right)\right)\left(\chi - \left(\frac{1 - \sqrt{5}}{2}\right)\right)}{(\chi - 1)(\chi + 1)} \leq 0$$



(9) Nature of Roots

Q If $D > 0$ then Root are Real?
yes!!

Q $x^2 + ix - 8 = 0$ find D

$$D = (i)^2 - 4 \times 1 \times -8$$

$$= -1 + 32 = 31 \oplus \text{Roots Real}$$

$$x = \frac{-i \pm \sqrt{31}}{2} \rightarrow \begin{matrix} \frac{\sqrt{31} - i}{2} \\ -\frac{\sqrt{31} - i}{2} \end{matrix} \rightarrow \text{Roots are not Real}$$

A) Nature of Roots depends on a, b, c & D .

B) a, b, c must be Rational